Optimal collision avoidance maneuvers using Pseudospectral Methods

European Space Surveillance Conference
7-9 June 2011, INTA

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ABSTRACT

The increasing concentration of space debris represents a rising threat to space security. This fact, along with the growing number of satellites in orbit, entails a higher probability of collision that leads as well to a higher number of Collision Avoidance Maneuvers (CAM). In this scenario, new methods to improve and optimize CAM need to be developed, in special when decision making processes about technology implementation are on board. Legendre and Chebyshev Pseudospectral (PS) Methods will be used to find out solutions to the optimal control problem stated in a CAM for low (continuous) thrust propulsion systems. A brief summary of the PS Methods for the optimal CAM calculation will be shown as integrated into a generic Operational Framework, and some results based upon typical values of a low-thrust propulsion system will be depicted.

KEYWORDS

Probability of collision, Collision Risk, Risk Assessment, Space Debris, Direct collocation methods, Optimal control, NLP, Pseudospectral Methods, Collision Avoidance Maneuver, Low-Thrust, Propulsion System, LEO.

GENERIC OPERATIONAL FRAMEWORK

One type of the Control Orbit Maneuvers is an emergency Collision Avoidance Maneuver (CAM) to be executed in case the probability of colliding with a space object (i.e. with space debris or with another satellite) is significant. In order to illustrate a more comprehensive representation of the whole problem, a generic Global Operational Framework is presented as a flow chart with different processes and procedures and it is exposed where the optimal CAM calculation is placed into this methodology as depicted in Fig. 1.

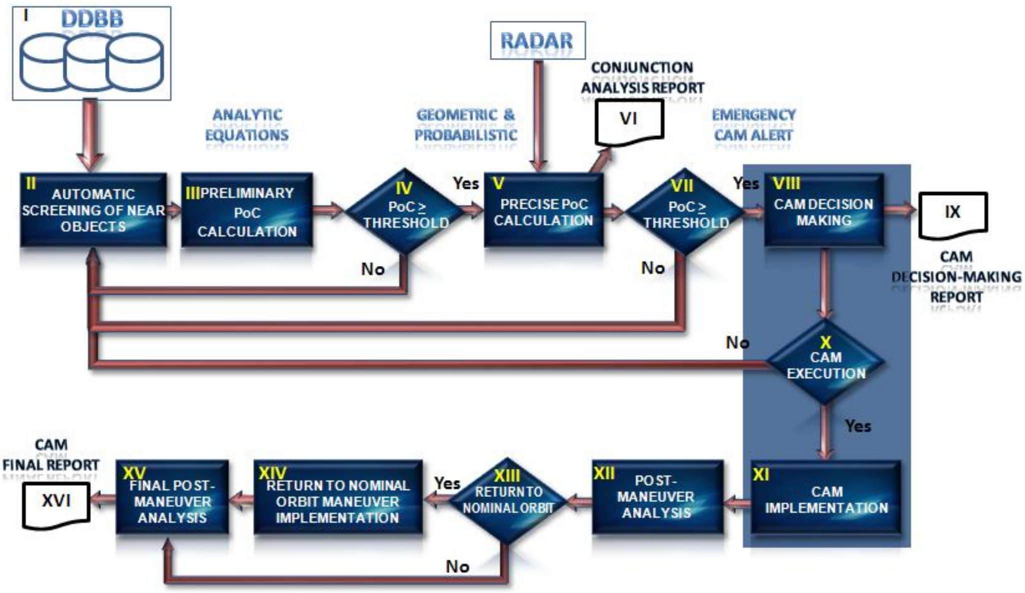


Fig. 1. LEO Satellite Collision Avoidance generic Operational Framework

An automatic screening of near objects (II) is made regularly using available databases (DDBB) such as ESOC DISCOS or USSTRATCOM (I). Analytic or basic equations are applied to the near objects for a preliminary Probability of Collision (PoC) calculation (III). In case a pre-established threshold is exceeded, a precise PoC calculation is done (V) and Conjunction Analysis Reporting is carried out (VI). Similarly, if the PoC exceeds a pre-established threshold (VII), an emergency CAM alert is raised that entails a Decision-Making process (VIII).

The evolution of PoC with time is an input of the Decision-Making process of a CAM, and the risk of performing the maneuver must be assessed taking into account several factors prior to the elaboration of an exhaustive Risk Analysis. The decision is taken using a Risk Acceptance Criteria as the upper limits of acceptable risks. In this step, the CAM and the possible come-back maneuver (return to the nominal orbit), have to be computed in order to get the data of the propellant usage, the remaining propellant, as well as its influence in the operational lifetime and cost. The maneuver uncertainties (low performance, thruster failure probability…) along with the possible mission degradation have to be included in the algorithms of the Multi-Criteria Decision Making (MCDM) process.

A CAM Decision-Making Report (IX) is generated and the decision of carrying out the maneuver is taken. If the CAM is decided according to the Risk Acceptance Criteria (X) and hence implemented (XI), a post-maneuver analysis is performed (XII) and afterwards has to be decided if a come-back maneuver to the nominal orbit is necessary (XIII).

To conclude with the process, after the final post-maneuver analysis (XV) a Final Reporting is conducted (XVI).

PSEUDOSPECTRAL METHODS

There are two approaches to solve an optimal control problem, the so called direct and indirect methods. In an indirect method, the first-order optimality conditions are derived using the Pontryagin's minimum principle and the calculus of variations. The optimality conditions lead to a Hamiltonian Boundary Value Problem (HBVP). On the other hand, Direct Methods -or Direct Collocation Methods- discretize the states and controls at points in time along the trajectory called nodes. Pseudospectral (PS) methods are a particular class of direct resolution methods for solving optimal control problems. In these methods, the states and control are approximated using a set of basis functions (usually Lagrange polynomials) which interpolate them at the selected collocations points and enforce the (dynamic and path) constraints at the N nodes over the time interval.

PS Methods are able to approximate continuous functions with an optimal number of nodes, resulting in lower-dimensional Nonlinear Programming (NLP) problems. These methods have an exponential (or spectral) convergence rate, that is to say, faster than any given polynomial rate, considering this feature as essential for solving complex optimal control problems.

There are several PS methods, but special attention will be paid in this paper to Legendre (see [6]) and Chebyshev methods (see [5], [12]). The most commonly used set of orthogonal collocation points in Legendre and Chebyshev PS methods are summarized in Table 1.

Table 1. PS Methods and orthogonal collocation points.

PS Method	Set of orthogonal collocation points	Nodes	Symmetry about the origin	Interval $\tau \in$
Legendre	Legendre-Gauss	$P_k(\tau)$	Yes	(-1,1)
	Legendre-Gauss-Radau	$P_{k-1}(\tau) + P_k(\tau)$	No	(-1,1] or [-1,1)
	Legendre-Gauss-Lobatto	$\dot{P}_{k-1}(\tau)$	Yes	[-1,1]
Chebyshev	Chebyshev-Gauss	$cos\left(\frac{2k-1}{2N}\pi\right)$	Yes	(-1,1)
	Chebyshev-Gauss-Radau	$cos\left(\frac{2(k-1)}{2N-1}\pi\right)$	No	(-1,1] or [-1,1)
	Chebyshev-Gauss-Lobatto	$cos\left(\frac{k}{N}\pi\right)$	Yes	[-1,1]

These sets of points are obtained from the roots of a Chebyshev or a Legendre polynomial and linear combinations of a Legendre polynomial and its derivatives. Applying an affine transformation (1), it is possible to shift the physical domain time t into a scaled time τ. The orthogonal collocation points are defined on the scaled-time domain $\tau \in [-1, 1]$.

$$\tau = \frac{2}{t_f - t_0} t - \frac{t_f + t_0}{t_f - t_0} \tag{1}$$

Where $P_k(\tau)$ denotes the k^{th}-degree Legendre polynomial.

As far as it is not the scope of this paper to reproduce the techniques to implement PS methods, it has been considered of interest to make a brief description before applying them to the determination of an optimal CAM.

OPTIMAL COLLISION AVOIDANCE MANEUVER FOR LOW-THRUST PROPULSION SYSTEMS

The determination of an optimal CAM problem using a continuous low-thrust system (non-impulsive), can be formulated as an optimal control problem using an appropriate cost function (Mayer, Lagrange or Bolza).

$$J = \Phi(x(t_0), t_0, x(t_f), t_f) + \int_{t_0}^{t_f} \mathcal{L}(x(t), u(t), t) dt \tag{2}$$

Where x is the state, u is the control, t is the time; Φ and $\mathcal{L}$ are the Mayer and Lagrange terms respectively; and the subscripts 0 and f refer to the initial and final conditions. According to this general equation, several particular cost functions can be established as shown in Table 2. A weighted-combined cost function can be formulated to analyze the convex mixture of several performance indexes.

Table 2. Example of cost functions for the optimal CAM problem.

Optimal maneuver problem Minimize	Problem	Cost function J	Variables
1. Time	Mayer	t_f	t_f: Maneuver duration
2. Fuel consumption	Mayer	m_f	m_f: Final mass
3. Total impulse	Lagrange	$\int_{t_0}^{t_f} T(t)\, dt$	$T(t)$: Thrust
4. Weighted combination	Bolza	$w_1 t_f + w_2 m_f + w_3 \int_{t_0}^{t_f} T(t)\, dt$	w_1, w_2, w_3 : Weights

In this paper, the optimal thrust under some constraints (case 3 of table 2) is derived using PS methods for LEO generic satellites dealing with a non-linear motion. In order to simplify the calculation some assumptions are taken for a coarse scenario:

a. The satellite is in the range of LEO
b. The satellite has a continuous low thrust directional propulsion system
c. The maneuver is non-impulsive and a constant exhaust velocity is applied
d. A point mass model is used
e. Only the CAM will be treated in the analysis (no return maneuver)
f. The mass consumption is very low in comparison with the mass of the satellite

The optimization problem tries to find the minimum of the J selected (optimal thrust in case 3), and can be drafted taking into account the following dynamic constraints, states and path constraint:

- Dynamic Constraints: It has been selected the simplified equations of motion for the coplanar orbit transfer (Keplerian equations) formulated in polar coordinates (r, θ):

$$\dot{r} = v_r \tag{3}$$

$$\dot{\theta} = \frac{v_n}{r} \tag{4}$$

$$\dot{v}_r = \frac{{v_n}^2}{r} - \frac{\mu}{r^2} + \frac{T_r}{m} \tag{5}$$

$$\dot{v}_n = -\frac{v_r v_n}{r} + \frac{T_n}{m} \tag{6}$$

$$\dot{m} = -\frac{\sqrt{T_r^2 + T_n^2}}{V_e} \tag{7}$$

Where v_r and v_n are the radial and normal components of the velocity; T_r and T_n are the components of the thrust and m is the satellite mass. The constants μ and V_e are gravitational parameter and the exhaust velocity respectively.

- States: Position (r and θ), velocity components (v_r and v_n) and mass of the satellite (m).
- Controls: Components of the thrust (T_r and T_n). The total thrust T is given by:

$$T = \sqrt{T_r^2 + T_n^2} \tag{8}$$

- Path Constraint: Maximum thrust (T_{max}) of the low-thrust propulsion system:

$$0 \leq T \leq T_{max} \tag{9}$$

Several variants of these equations can be derived as in [13].

SIMULATION RESULTS AND ANALYSIS

PS Methods can be applied to a wide variety of space optimal control problems and more concretely to the determination of optimal orbit maneuvers. In this section, examples of the simulation results and analysis of the optimal CAM problem with the goal of minimizing the total impulse for a LEO low thrust propulsion system satellite using PS methods are presented.

The problem is solved using non-dimensional forms of the equations that are partially based on canonical units as in [13], [14]. Dimensionless thrust can be expressed as:

$$T^* = \frac{Ta^2}{\mu m} \tag{10}$$

Where a is the initial semi-major axis. The mass is considered as constant to avoid the use of dimensional parameters of the satellite like V_e in (6), and the cost function to minimize the total impulse provided to the satellite to perform the CAM is:

$$J = \int_{t_0}^{t_f} T^*(t)\, dt \tag{11}$$

The dimensionless maximum thrusts considered for the examples are shown in Table 3, along with a corresponding range of approximate thrust values (mN) for a 100 kg Little LEO and a 1.200 kg Big LEO and orbits from 400 km to 2.000 km. It must be remarked that the optimal CAM problem is solved for low-thrust propulsion systems.

Table 3. Dimensionless maximum thrusts considered and its approximate values for LEO satellites.

Dimensionless T_{max}	**T (mN)**	
T^*_{max} x 10^4	Little LEO 100 kg	Big Leo 1.200 kg
0,125	7-11	85-130
0,250	14-22	171-261
0,500	28-43	341-522
1,000	57-87	683-1043

In Fig. 2 is shown the CAM general geometry.

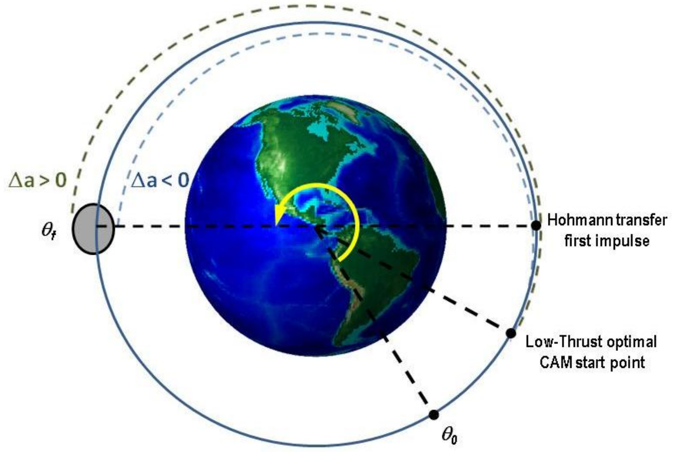


Fig. 2. CAM general geometry (Δa stands for the increment of the semi-major axis).

The initial anomaly to solve the problem is given by θ_0 and the final anomaly is given by the estimated point of closest approach with the secondary object θ_f (i.e. orbital debris). The initial and terminal conditions of r are (using dimensionless variables):

$$r_0 = 1; r_f = 1 - 2\Delta a^* \tag{12}$$

Where Δa^* is the required dimensionless increment of the semi-major-axis in the CAM. The size of the maneuver can be calculated using the following equation:

$$|\Delta a| = f\left(\frac{n(\sigma_P + \sigma_S) + (R_P + R_S) - D_{PS}}{2}\right) \tag{13}$$

Where:

f : Factor to take into account uncertainties (maneuver performance)
n : number of sigmas (equal number for primary and secondary objects)
σ_P : radial error of the primary object –LEO satellite- (standard deviation)
σ_S : radial error of the secondary object –i.e. orbit debris- (standard deviation)
R_P : radius of the primary object (geometric parameter)
R_S : radius of the secondary object (geometric parameter)
D_{PS} : estimated distance of closest approach

If the estimated height of the secondary object (in the closest approach) is higher than the satellite height, then the semi-major axis increment will usually be negative and vice-versa.

A set of optimal CAM varying the maximum non-dimensional thrust for a given magnitude of the maneuver ($\Delta a^* = 7.14 \times 10^{-5}$) has been computed. The initial condition θ_0 is $-\pi$ (one period before the closest approach). The non-dimensionalized components of the thrust (T_r^* and T_n^*) and the start and end points of the optimal CAM are shown in Fig. 3.

It can be noticed that as the value of the maximum thrust is higher, the optimal CAM approaches more to a first impulse of a Hohmann transfer maneuver placed half an orbit before the estimated closest approach to the secondary object (θ = 0º). However, if T^*_{max} decreases, the optimal CAM maneuver starts earlier and ends later.

In the Fig. 4 the thrust direction and its relative magnitude is illustrated with arrows for the optimal CAM with increment and decrement of the semi-major axis. In this graph, the maximum value of the non-dimensional thrust is equal to 0.125×10^{-4}. The component T_r changes of sign in the midway point of the maneuver.

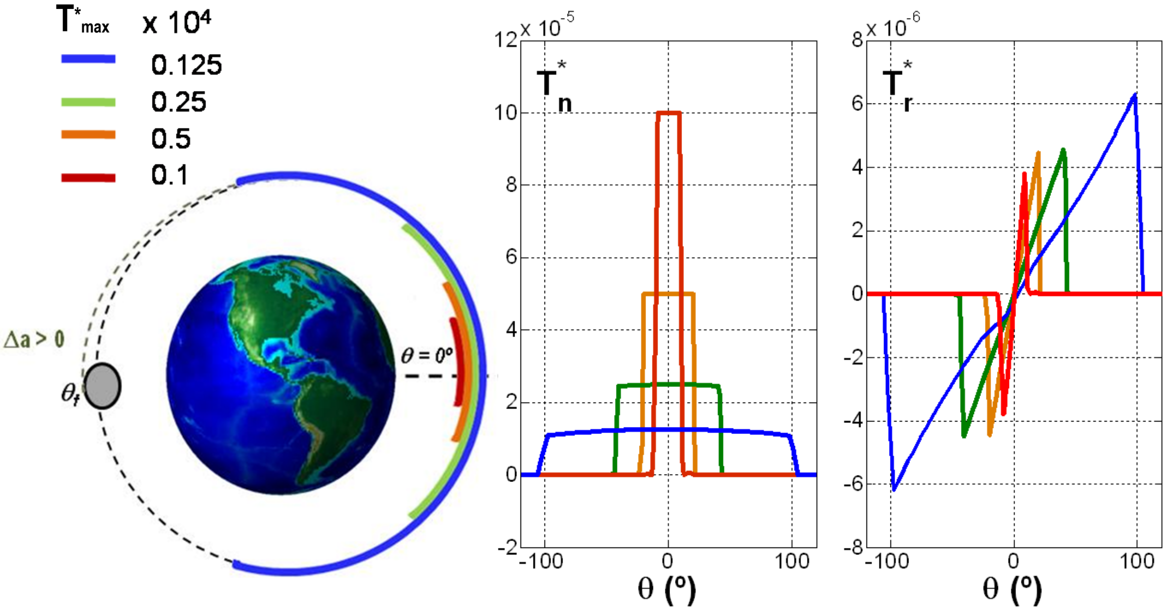


Fig. 3. Dimensionless thrust components and start and end points for optimal CAM related to different values of T^*_{max}.

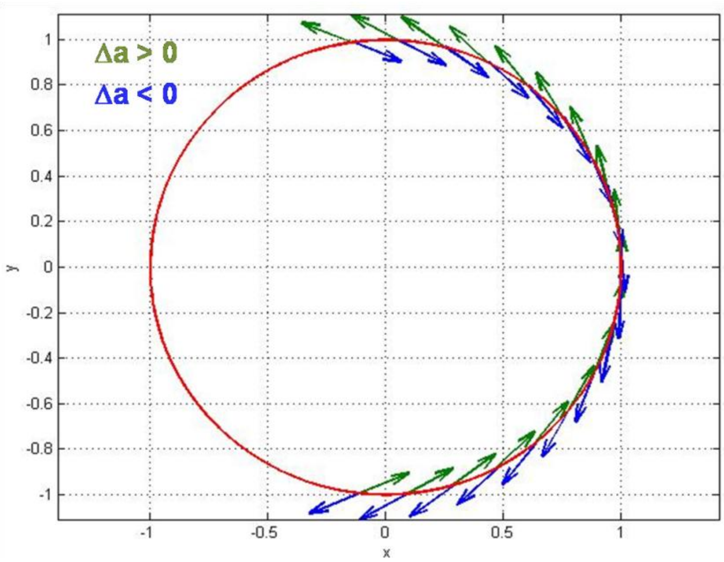
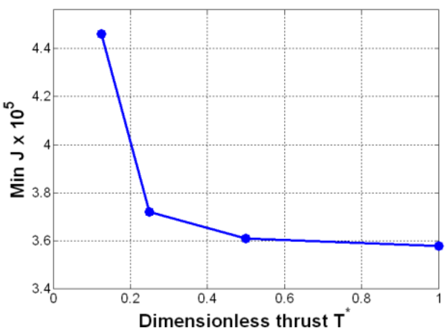


Fig. 4. Thrust relative magnitude and angle profile for optimal CAM with increment and decrement of the semi-major axis.

In the Table 4 the results of the minimum cost function (minimum non-dimensional impulse) are shown. The impulse for the CAM using a first impulse of a Hohmann transfer has been included in the last file. The minimum cost function approaches to that of the first impulse of a Hohmann transfer as the dimensionless thrust is higher.

Table 4. Minimum cost functions for the optimal CAM for different dimensionless thrusts (Δa^* =7.14x10^{-5})

Dimensionless *T* ***T_{max}* x 10^4**	**Cost function** ***Min J***	**Maneuver** ***$\Delta\theta = \theta_{end} - \theta_{start}$ (º)***
0,125	4.46 x 10^{-5}	205
0,250	3.72 x 10^{-5}	88
0,500	3.61 x 10^{-5}	43
1,000	3.58 x 10^{-5}	21
Hohmann first impulse	3.57 x 10^{-5}	Impulsive



In Fig. 5, a set of optimal CAM has been depicted varying the increment of the semi-major axis for a given value of the maximum dimensionless thrust (T^*_{max} = 0.25 x 10^{-4}). The increment of the semi-major axis is given in meters for an orbit height of 630 km. As can be seen, as the size of the maneuver decreases, the duration of the optimal CAM is lower.

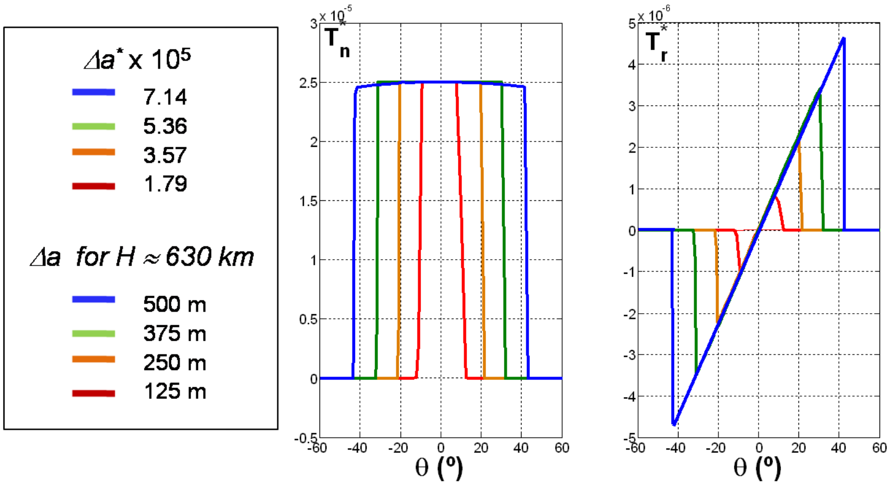


Fig. 5. Optimal CAM varying Δa^* for T_{max}^* = 0.25 x 10^{-4}.

Both Legendre and Chebyshev PS methods have been used to find the optimal CAM, concluding that the results and computational cost are quite similar. The optimal CAM calculated in these examples was implemented and tested in an E&Q's orbital dynamics simulator in order to check its correctness.

SUMMARY AND CONCLUSIONS

Summary

In this paper, a generic Operational Framework has been briefly depicted; CAM planning process is included into an operational framework which takes into account several issues like MCDM block, Risk Assessment and many operational blocks. An introduction to the PS methods, considering different basis functions and sets of orthogonal collocation points, as well as the application of these direct methods to the computation of optimal CAM with low-thrust continuous propulsion has been presented for a selected scenario (LEO satellites, circular orbit and keplerian equations for the transfer orbit). As an example of application, a set of simulations for different maximum thrusts and magnitudes of CAM for the non-linear motion LEO satellite has been solved using PS methods. The results of the optimal maneuvers have been analyzed and represented graphically drawing some conclusions.

Conclusions

The basic idea of this study is to highlight the power of using PS methods for the direct resolution of the optimal control problem stated in a CAM, just in case a low thrust propulsion system is available for a generic LEO satellite. A few performance indexes (cost functions) were illustrated and the thrust of the optimal CAM was calculated as an example, showing the following conclusions:

a) This study experiences the use of PS methods to demonstrate the capability to assess a technology performance based on an optimal CAM.
b) Examples of the optimization of continuous low-thrust CAM have been set up and solved.
c) The performance index (minimum impulse) of the optimal CAM using low-thrust is quite similar (slightly higher) to the equivalent impulsive maneuver (Hohmann transfer first impulse half an orbit before the estimated closest approach to the secondary object).
d) The mass involved in the transfer orbit could be calculated for some specific impulses (i.e. electromagnetic and chemical) in order to get a technology comparison.

Further work can be done for CAM using the same core as employed in this paper for the following issues:

- Investigate the advantages of the employment of low-thrust propulsion systems in terms of mass saving and the associated cost
- Take into account satellite attitude and drag constraints for low LEO CAM
- Consider combined propulsion systems dealing with complex scenarios (Space Vehicles)
- Analyze and integrate multiple criteria for the CAM Decision Making Process, combining this criteria with the ones included in performance indexes (see Bolza problem in table 2)

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